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The Effectiveness of Digital Marketing Channels for Customer Acquisition

Abstract

This paper investigates the intertemporal dynamics between paid display ads and organic traffic, focusing on their effects on new customer acquisition. The study uses a structural vector autoregression (SVAR) approach with aggregate-level daily data over 523 days to explore the interaction effects of Facebook display ad impressions and organic sessions in a small pet food business over time. Key results indicate that no statistically significant positive synergy is found between paid display ads and organic traffic. Furthermore, while display ads demonstrate a sustained positive impact on sales, organic traffic exhibits a strong initial impact on sales that quickly diminishes over time.

Plagiarism Declaration

I confirm that this is entirely my own work and has not previously been submitted for assessment, and I have read and understood the University's and Faculty's definition of Plagiarism.

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(6186 words + 4 pages of table + 7 equations)

1. Introduction

The global digital marketing market is expected to reach \$667 billion in 2024, with a projected compound annual growth rate of 9% through 2026 (Marketing Report, 2023). A report by Gartner (2023) reveals that 72% of overall marketing budgets are dedicated to digital marketing channels. Given the growing impact and importance of digital marketing, a critical challenge faced by businesses, particularly small online enterprises, is the efficient allocation of marketing budgets to convert customers at minimal cost. Marketing attribution modelling thus emerges as a crucial tool in this context.

Romero Leguina et al. (2020) defines an attribution model as “a set of rules to attribute the success of a conversion across different marketing events”. Attribution models are helpful for marketers to assess the real returns to investment in different channels and campaigns. However, isolating the effect of specific marketing activities in multi-channel marketing is difficult. Key challenges that confound the analysis in multi-channel marketing environments include marketing carryover, interaction, and spill-over effects, detailed in Table 1 below.

Table 1: Concepts in multi-channel marketing

Concept	Description	Reference
Interaction Effect	Impact of one marketing channel is influenced by another, showing non-additive interdependencies.	Danaher and van Heerde (2018)
Carryover Effect	Impact of marketing actions that extends beyond the immediate period.	Breuer and Brettel (2012)
Spill-over Effect	Traffic in one marketing channel leads to visits and conversions via another channel	Li and Kannan (2014)

This study specifically focuses on addressing the intertemporal interaction effect between paid display ads and organic traffic. This paper uses aggregate-level daily data from a small online pet food business spanning a period of 523 days to analyse the intertemporal interaction effect of Facebook display ad impressions and total organic sessions on new customer acquisition. Using a structural vector autoregression (SVAR) approach, this study finds no statistically significant synergy effect of paid display ads impressions on the volume of organic traffic to the business’s website. The impulse response analysis also reveals that there is a distinct pattern in the response of new customer acquisition to the two channels:

Facebook display ad impressions shows a sustained positive impact on sales, while organic traffic shows a strong initial impact that diminishes over time. The results have significant managerial implications and highlight the need for marketers to adopt a dynamic approach in budget allocation, one that considers not only the immediate returns but also the longer-term effects. The SVAR analysis detailed in this paper also provides a straightforward and aggregate-data friendly way for small businesses to re-evaluate their marketing strategies.

2. Review of Literature and Conceptual Framework

2.1 Background of Marketing Attribution Models

Marketing attribution in digital marketing has evolved to address the complexities due to the increasing prevalence of multi-channel strategies. Early attribution models such as the first- or last-click model attributes the conversion exclusively to only one specific touchpoint (Jordan *et al.*, 2011). Other widely adopted models give credit uniformly among all the touchpoint exposures received by a customer, or give a higher weight to more recent exposures (Garina, 2023). These models, however, make overly simplistic assumptions about customer behaviours and thus lead to inefficient marketing decisions. In response to these limitations, recent scholarly efforts have moved towards more sophisticated, data-driven approaches. Modern attribution models incorporates methods and algorithms such as the Shapley value, neural networks, Markov chain and econometrics modelling to analyse data in both converting and non-converting paths (Romero Leguina *et al.*, 2020) and try to capture the spill-over, carryover and interaction effects in multi-channel marketing.

2.2 Theoretical Foundations and Empirical Evidence

The attribution problem of interest in this study is the dynamic interactions between paid display ads and organic traffic¹ in driving sales. Gatignon and Hanssens (1987) proposed that “marketing efforts create sales synergistically rather than independently”, laying the ground for the importance of modelling interactions. The concept of marketing synergy between different types of channels has been discussed extensively in marketing theories. A study by Çizmeçi and Ercan (2015) incorporates normative inferences through interviews and found that paid digital marketing content effectively increases brand awareness. Marketing response models proposed by Özçifçi (2017) suggest that increased brand awareness and visibility are the basis for purchase intention, which is closely tied with organic customer interest. Exposure to passive forms of advertising, such as display ads, may also influence consumers’ consideration sets and potentially moves the consumers down the funnel towards direct

¹ Organic traffic refers to visitors that arrive at a business’s website from unpaid sources. For example, through direct navigation, referrals, or social media posts without any paid promotion.

engagement (Romero Leguina et al., 2020). These works lay the theoretical foundation of how paid advertising can have a lasting and amplifying effect on direct sessions and other forms of organic traffic.

Empirical studies have sought to verify and quantify such relationship. Yang and Ghose (2010) studied the interdependence of paid search and organic search using Markov chain Monte Carlo methods, and found that positive synergy exists in both directions between these two channels. Ghose and Todri-Adamopoulos (2016) extends this analysis with a quasi-experiment research design. They use the Difference-in-Difference method to analyse the effect of an exogenous shock to viewability of display ads, and concludes that mere exposure to display ads increases users' propensity to search for the brand and its products. Nottorf (2014) shows an interesting contrasting result that for more than 90% of the consumers, repeated exposure to display ads decreases their search click probabilities. These studies provide strong empirical evidence suggesting that paid digital marketing and organic channels interact in a meaningful way that affects customer behaviour and business outcomes. Building on this foundation, this paper further explores and tests the structure, direction, and magnitude of the intertemporal interactions between paid marketing, organic traffic, and sales. I expect to find dynamic synergy between paid display ads and organic sessions that effectively contributes to new customer acquisition.

2.3 Gaps in Current Literature

De Haan, Wiesel and Pauwels (2016) used the SVAR method to explore the relative long-term effectiveness of nine forms of social media and search engine marketing. They identified how long the impacts last and where the impacts are rooted in the conversion funnel. However, the existing literature about the *interactions between digital marketing display ads and organic traffic* predominantly focuses on the contemporaneous or immediate impact. There remains a significant gap in understanding the duration of such interaction, and the pattern of how they evolve overtime. This study adopts a similar SVAR approach to address this gap.

Furthermore, empirical studies on multi-channel marketing attribution often cater to larger businesses, focusing on broad trends that may not align with the unique challenges and

opportunities faced by small businesses. Larger businesses generally operate with more marketing budgets and resources, allowing them to access a wider range of analysis tools and strategies. Existing literature also tends to rely on individual-level path data, which requires extensive and complex datasets. Such datasets are particularly hard for small businesses to obtain, because of privacy regulations, technical complexities and the significant resources required for data management. This paper helps to address these challenges by introducing a methodology that uses the more accessible aggregate-level data to analyse the temporal interactions between displaced ads and organic traffic.

3. Data Description

The dataset for this study was acquired through a partnership with Charlie Oscar, sourced from an online pet food business. The business operates primarily on its own website with a focus on the premium fresh dog food market. The dataset comprises daily records of a range of marketing and customer acquisition metrics spanning January 2022 to June 2023, covering 523 days. These metrics can be categorised as follows:

- Digital marketing impressions and clicks: statistics on impressions and clicks from both Facebook (Meta) and Google advertising campaigns.
- Website traffic sources: statistics on how users arrive at the website organically, whether through direct entry, organic search, email campaigns, referrals, or organic social media posts.
- Sales metric: statistics on daily number of new customers signed up to the business.

Table 2 shows all primary data observations and their detailed descriptions.

Table 2: Observations in primary data

Observations	Description
New Customers	Influx of new customer to the business, recorded by the ecommerce tracking system on the date of signup.
FB Prospecting Impressions	Total count of impressions generated on Facebook ads targeting potential new customers unfamiliar with the brand.
FB Retargeting Impressions	Total count of impressions generated on Facebook ads targeting users who have previously engaged with the brand.
FB Influencer Prospecting Impressions	Total count of impressions generated on Facebook ads targeting potential new customers through the endorsement of influencer content.
FB Influencer Retargeting Impressions	Total count of impressions generated on Facebook ads targeting users who have previously engaged with the brand through the endorsement of influencer content.
Google Brand Clicks	Volume of user clicks on brand-specific Google ads initiated through searches of the specific brand name.
Google DSA Clicks	Volume of user clicks on Dynamic Search Ads (DSA) on Google, automatically generated based on the content of the business's website.

Google Generic Clicks	Volume of user clicks on Google ads triggered by search terms not directly associated with the brand name, but with broader relevant terms.
Google Pmax Clicks	Volume of user clicks received from Google’s Performance Max campaigns, reflecting cross-channel Google ads.
Direct Sessions	Website visits initiated by users directly entering the URL or accesses via a bookmark.
Email Sessions	Website visits initiated by user engagement with links embedded within email marketing campaigns.
Organic Search Sessions	Website visits initiated from unpaid or organic search engine results.
Referral Sessions	Website visits initiated from links on external websites.
Organic Social Sessions	Website visits initiated from unpaid social media posts.

Note that one limitation of this dataset is that we only have aggregate data of customer acquisition without access to granular individual-level path data. Specifically, the dataset lacks insight about customer’s journey towards conversion and thus cannot precisely attribute actions and conversions to specific marketing touchpoints. However, with aggregate data, we are able to gain a better understanding on the overall trends and patterns in customer behaviour, providing insights for marketing decision-making at a macro level.

4. Methodology

4.1 Modelling Approach and Justification

Traditional OLS or other single-variable time series models fall short in this study because they do not handle jointly determined variables well and fail to capture the intertemporal interactions among variables. Instead, a systems regression model is required. In particular, Vector Autoregression (VAR) models allow researchers to “use a systems approach to explain the multiple channels of influence of marketing variables on each other” (Srinivasan, 2022). VAR models addresses endogeneity by incorporating lagged values of all variables to capture the complex feedback loops (Dekimpe and Hanssens, 2007). Therefore, I choose to use a VAR model to analyse the dynamic relationships among the variables of interest.

Another important consideration when applying the VAR method to the context of this study is that not all temporal effects are feasible, given the sequence of a marketing funnel progression. For instance, organic traffic should not affect the display ad impressions contemporaneously as paid ads are mostly predetermined by marketing budgets and the practices of the media platforms. Thus, we need a restricted model to test certain specified interactions that is plausible based on marketing theories and practical limitations. A Structured VAR (SVAR) model which imposes a priori constraints to reflect a sequence of events is therefore the natural next step. Although SVARs are predominantly used in the field of macroeconomic analysis (Gottschalk, 2001), they are appropriate to use in this context as well because SVAR models are “specifically designed to supplement sample-based information with managerial judgment and/or marketing theory” (Dekimpe and Hanssens, 2000).

4.2 Data Preparation

I choose to include three endogenous variables and a set of exogenous variables in the VAR model, as detailed Table 3 below:

Table 3: Variables in the VAR model

Variable Name	Description	Derivation from Primary Data	
Endogenous	<i>FB</i>	Total Facebook impressions	Sum of (FB Prospecting Impressions, FB Retargeting Impressions, FB Influencer Prospecting Impressions, and FB Influencer Retargeting Impressions) divided by 1000
	<i>OS</i>	Total organic sessions	Sum of (Direct Sessions, Email Sessions, Organic Search Sessions, Referral Sessions, and Organic Social Sessions) divided by 10
	<i>NC</i>	Number of new customers acquired	Direct count of new customers
Exogenous	<i>Gclicks</i>	Total Google clicks	Sum of (Google Brand Clicks, Google DSA Clicks, Google Generic Clicks, and Google Pmax Clicks)
	<i>Weekdays</i>	Dummy variables for day-of-the-week	-

A key consideration in choosing the endogenous variables in the VAR model is the trade-off between the degrees of freedom and the number of variables studied. The number of parameters in the reduced form VAR grows in the square of the number of endogenous variables. Specifically, if we study to study t lags with n endogenous variables, then there will be $n + pn^2 + \frac{n(n+1)}{2}$ parameters. To balance model simplicity and explanatory power, I choose to simply aggregate the metrics of the same categories. This consideration leads to a total of three endogenous variables: new customers (*NC*), aggregated total paid sessions (*FB*) and aggregated total organic sessions (*OS*) respectively.

For the paid channel of interest, I choose to use total Facebook impressions over Google for a few reasons. Firstly, as the business’s main marketing tool, *FB* contributes to the most to total number of impressions, and spends the largest share of the social media marketing budget. Secondly, aggregating Facebook impressions makes more sense because the four categories of Facebook marketing metrics are similar in nature. In contrast, marketing campaigns on Google are more heterogeneous in targeting and strategy, making aggregation and later interpretation of the results challenging. A linear transformation is performed to scale down total *FB* impressions by 10^3 and ensure all the endogenous variables have a

similar order of magnitude. For the organic channel, I aggregate the five categories of organic channel sessions and use the total organic sessions.

Other variables from the primary dataset are included as exogenous variables. They are controlled for in each period while not having any intertemporal effects within the model. To control for potential variations in marketing effectiveness and customer behaviours, I have also included six day-of-the-week dummies. Table 4 provides summary statistics of the variables.

Table 4: Summary statistics

	<i>FB</i>	<i>OS</i>	<i>NC</i>	<i>Gclicks</i>
Mean	160.9155	790.8719	34.41683	320.0803
Median	162.0860	798.0000	34.00000	311.0000
Maximum	336.6580	1522.000	77.00000	995.0000
Minimum	39.44300	267.0000	5.000000	29.00000
Std. Dev.	59.63185	226.9252	12.88641	144.2896

4.3 Model Estimation Steps

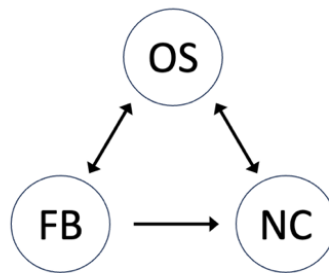
Table 5 illustrates an outline of the modelling steps I took in this study, adapted from the illustrative approach to VAR modelling framework suggested in Srinivasan (2022).

Table 5: Outline of model estimation steps

	Research Goal	Methodological Step	Relevant literature
1	Granger causality test	Granger causality	(Granger, 1969)
2	Unit root and cointegration tests	Augmented Dickey-Fuller (ADF) test Cointegration test	(Dickey and Fuller, 1979) (Kwiatkowski <i>et al.</i> , 1992) (Johansen, 1991)23/04/2024 23:28:00
3	Model of the dynamic system	Vector autoregression model (VAR) Structured VAR model	(Dekimpe and Hanssens, 1999) (De Haan, Wiesel and Pauwels, 2016)
4	Simulation and analysis	Restricted impulse response analysis	(Pesaran and Shin, 1998) (Pauwels, Hanssens and Siddarth, 2002)
5	Drivers of performance	Forecast variance error decomposition (FEVD)	(Hanssens, 1998)

In step one, pair-wise Granger causality tests are set up for the three variables of interest: *FB*, *OS* and *NC*. Figure 1 shows that all variables are connected by Granger-causal relationships and thus should enter the system of persistence modelling as endogenous. In the figure, arrows between variables indicate the direction of causal relationship between them. For instance, the double arrow between *NC* and *OS* indicates that total organic sessions and new customers acquired is found to Granger-cause each other. Figure 1 also presents a model-independent suggestion that Facebook impressions (*FB*) may both have an impact directly on new customer acquisition (*NC*), and indirectly through its impact on organic sessions (*OS*). The presence of this dual pathway indicates the presence of a complex dynamic relationship and justifies the SVAR approach.

Figure 1: Granger causality relationships between endogenous variables



In step two, ADF tests are performed to determine if the endogenous variables are stationary, which is a prerequisite for VAR model estimation. Table 6 shows the result of the unit root test, where all variables are shown to be difference-nonstationary but trend-stationary. To address this, I incorporate a time trend as an exogenous variable within the VAR model to control for any deterministic trend in the data, such as a steady increase in brand recognition or customer loyalty over time. Given the trend stationary nature of the variables, cointegration tests are no longer necessary because the series is not integrated of order 1.

Table 6: Summary of ADF test statistic

	<i>FB</i>	<i>OS</i>	<i>NC</i>
ADF with constant	-2.488	-2.159	-2.502
ADF with constant and linear trend	-3.267	-11.959	-3.181

Note: bold numbers indicate significant evidence of non-stationarity.

In step three, based on the outcomes of the Granger causality and unit root test, we specify a three-variable VAR model that describes the dynamic relationships among total Facebook

impressions (FB), total organic sessions (OS) and new customers acquired (NC), while controlling for total Google clicks ($Gclicks$), day-of-the-week and a time trend. A few selection criteria for the optimal lag order of our model are presented in Table 8 in the Appendix. Subsequent Lagrange Multiplier (LM) tests for autocorrelation indicated that 4 is the minimum number of lags to eliminate autocorrelation in residuals and ensure a robust model specification. Thus, I followed the Akaike Information Criterion (AIC) to use 4 lags in the model.

Before imposing of any temporal ordering, the underlying VAR model in structural form is therefore specified as:

$$\begin{bmatrix} \alpha_{11}^0 & \alpha_{12}^0 & \alpha_{13}^0 \\ \alpha_{11}^0 & \alpha_{12}^0 & \alpha_{13}^0 \\ \alpha_{11}^0 & \alpha_{12}^0 & \alpha_{13}^0 \end{bmatrix} \begin{bmatrix} FB_t \\ OS_t \\ NC_t \end{bmatrix} = \begin{bmatrix} \gamma_1^0 \\ \gamma_2^0 \\ \gamma_3^0 \end{bmatrix} + \sum_{i=1}^4 \begin{bmatrix} \beta_{11}^i & \beta_{12}^i & \beta_{13}^i \\ \beta_{11}^i & \beta_{12}^i & \beta_{13}^i \\ \beta_{11}^i & \beta_{12}^i & \beta_{13}^i \end{bmatrix} \begin{bmatrix} FB_{t-i} \\ OS_{t-i} \\ NC_{t-i} \end{bmatrix} + \psi X_t + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \quad (1)$$

Or, in matrix notation for simplicity:

$$AY_t = C + \sum_{i=1}^4 B_i Y_{t-i} + \psi X_t + e_t \quad (2)$$

where Y_t is a 3×1 vector of endogenous variables, C is a 3×1 vector of intercepts, X_t and ψ are the vector of exogenous variables and parameters, and e_t is the residual matrix.

Matrices A and B_i are what we are most interested in estimating. A is a 3×3 matrix of parameters that shows the *contemporaneous* effects, while B_i 's is 3×3 matrices of parameters that shows the *lagged* effects among the endogenous variables.

It is important to note that, however, without imposing any structural restrictions we are only able to estimate the reduced form VAR, which is a transformed version of the true model that contains less information:

$$Y_t = C' + \sum_{i=1}^4 B_i' Y_{t-i} + \psi' X_t + e_t' \quad (3)$$

$$\text{where } A^{-1}C = C' \text{ and } A^{-1}B_i = B_i'$$

From the reduced form VAR, we are able to get estimates for C' and B_i' . In order to uniquely pin down matrices A and B , further information is required to recover the structural VAR.

4.4 SVAR Specification

To recover the SVAR, I use the Cholesky identification scheme which could be thought of as imposing a causal ordering on the endogenous variables. Based on theoretical predictions about the marketing funnel, I establish the causal sequence as FB , OS , and finally NC .

Total Facebook Impressions (FB) is placed first because Facebook campaigns are designed to seed interest and brand awareness, usually predetermined by business's budget decisions. So it is likely not influenced by fluctuations in organic traffic or new conversions on the same day. Organic session (OS) is placed second, as it can only be contemporaneously affected by FB but not NC . Increased visibility from FB could lead to more organic website visits within the same period. Finally, new customer acquisition (NC), as the outcome variable, is placed last, as FB and NC both contribute to sales starting from the contemporaneous period.

For a three-variable VAR model, 6 restrictions are required to derive the structured form. The causal ordering of $FB \rightarrow OS \rightarrow NC$ is imposed, which is to set A as a lower-triangular matrix and require and require the error terms $\varepsilon_{i,t}$ are uncorrelated with each other.

$$\text{Let } A = \begin{bmatrix} \alpha_{11}^0 & \alpha_{12}^0 & \alpha_{13}^0 \\ \alpha_{11}^0 & \alpha_{12}^0 & \alpha_{13}^0 \\ \alpha_{11}^0 & \alpha_{12}^0 & \alpha_{13}^0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{11}^0 & 1 & 0 \\ \alpha_{11}^0 & \alpha_{12}^0 & 1 \end{bmatrix}$$

The structural form in (1) can thus be written as:

$$\begin{bmatrix} 1 & 0 & 0 \\ \alpha_{11}^0 & 1 & 0 \\ \alpha_{11}^0 & \alpha_{12}^0 & 1 \end{bmatrix} \begin{bmatrix} FB_t \\ OS_t \\ NC_t \end{bmatrix} = \begin{bmatrix} \gamma_1^0 \\ \gamma_2^0 \\ \gamma_3^0 \end{bmatrix} + \sum_{i=1}^4 \begin{bmatrix} \beta_{11}^i & \beta_{12}^i & \beta_{13}^i \\ \beta_{11}^i & \beta_{12}^i & \beta_{13}^i \\ \beta_{11}^i & \beta_{12}^i & \beta_{13}^i \end{bmatrix} \begin{bmatrix} FB_{t-i} \\ OS_{t-i} \\ NC_{t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} + \psi X_t \quad (4)$$

After applying these restrictions, we can recover the SVAR and get estimates for the B matrices using simple matrix transformation. Following the SVAR recovery, I proceed to do an impulse response analysis to simulate shocks to FB and OS , and trace out the effect of them on the system. Impulse-response graphs can trace the effects of a one-standard-deviation shock to FB and OS on NC over time, providing insights into the intertemporal dynamics of marketing channels.

5. Results

5.1 VAR Model Fit

Table 9 in the Appendix shows the full set of estimation results for all 63 parameters in the VAR model, which forms the elements of matrices B_1' to B_4' . They inform us about how the previous 4 lagged periods of the variables predict the current period values. Recall the reduced form VAR estimates:

$$Y_t = C' + B_1'Y_{t-1} + \dots + B_4'Y_{t-4} + \psi'X_t + \Sigma_t'$$

Several diagnostic tests are conducted post-estimation to ensure validity of the model. The model exhibits good fits, with a R^2 statistics of 0.878. The Lagrange Multiplier (LM) test fails to find significant evidence of residual autocorrelation, and all roots of the characteristic polynomial lies inside the unit circle indicating that the model is stable. The Jarque-Bera test fails to reject that the residuals follow a normal distribution. This also validates my previous decision for data aggregation and transformation as they appear to not have adversely affected the distributional characteristics of the data. The model is thus found to be well-specified.

As seen from Table 9, the presence of significant coefficients for lagged values of FB , OS , and NC implies the intertemporal interactions among paid marketing efforts, organic search sessions, and new customer acquisitions. We can also infer from the significant coefficients on the exogenous variables that accounting for weekday patterns and time trend do help to explain some external fluctuations in this model.

To read the coefficients, for example, $FB(-1)$ as a coefficient of 0.688 on FB with a standard error of 0.047, which suggests that the immediate past period Facebook impressions have a statistically significant positive effect on the current value of FB in the VAR model. This aligns with our expectation.

More potential inferences could be made on the results. However, as mentioned before, reduced form VAR estimates does not account for the contemporaneous relationships between endogenous variables. They are not optimal for drawing any definitive causal

interpretations. We thus conclude from the initial VAR analysis that the data demonstrates the desired intertemporal patterns, and now proceed with the SVAR analysis.

5.2 SVAR Results

5.2.1 Contemporaneous Effects

Table 7 shows the estimates of parameters in matrix A . The matrix form representation is:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -0.340 & 1 & 0 \\ -\mathbf{0.036} & -\mathbf{0.023} & 1 \end{bmatrix}$$

Table 7: SVAR estimates for matrix A

	Coefficient	Std. Error	z-Statistic
Effect of <i>FB</i> on <i>OS</i>	-0.339903	0.243192	-1.397675
Effect of <i>FB</i> on <i>NC</i>	-0.035712	0.014860	-2.403249
Effect of <i>OS</i> on <i>NC</i>	-0.023288	0.002677	-8.699077

Note: Bolded coefficients are statistically significant at 1% level

Effect of *FB* on *OS*

Matrix A describes the contemporaneous relationships among the endogenous variables. The coefficient of -0.340 suggests a positive but modest contemporaneous effect of *FB* on *OS*. Specifically, a 1000-unit increase in total Facebook impressions is associated with a 0.340 unit increase in total organic sessions in the same period, *ceteris paribus*. In other words, a new customer arrives through organic traffic every 3,000 increase in total Facebook impressions (given the average volume in that period is 160,000). This demonstrates evidence for some contemporaneous positive synergy effect of Facebook display ads on organic traffic.

However, the p-value (0.162) indicates that this relationship is not statistically significant. The lack of significance could be attributed to the relatively small sample size. The sample size to parameter ratio stands at 8.3, which does approach a threshold that could hamper the model's ability to detect smaller but meaningful effects. Ideally, a larger dataset could lead to more robust results and provide more definitive conclusion.

Effect of *FB* and *OS* on *NC*

The coefficients -0.036 and -0.023 suggest that a 1000-unit increase in Facebook impressions is associated with an increase of 0.036 units in new customers, and a unit increase in organic sessions is associated with a 0.023 unit increase in new customers. Both relationships are statistically significant. This confirms that both paid social media ads campaigns and organic sessions directly drive new customer acquisitions in the same period, which aligns with what we expect from the literature.

5.2.2 Lagged and Net Effects

The estimated coefficient matrices that describe the structural lagged effects can be derived from linear transformations of the VAR model matrices estimated. However, the resulting $12 \times 3 \times 3$ matrices are difficult to describe or summarise in a neat and accurate way. Moreover, the complex feedback loops and interdependencies across equations further complicates the interpretation issue.

Therefore, for lagged effect analysis, I follow Sims (1980)'s recommendation that in multivariate time series models, "the best descriptive device appears to be analysis of the system's response to typical random shocks". Typical random shocks refer to the residuals of one standard deviation unit in each equation of the system (Sims, 1980). Another way to view the impulse response functions (IRFs) is that it models the difference between two forecasts: one that is based on the information set that does not take the shock into account, and the other that predicts based on an extended information set that takes the shock into account (Srinivasan, 2022). I use impulse response analysis as the main tool to analyse the impact of display ads on organic traffic, and the impact of display ads and organic traffic on new customer acquisition.

I constructed the IRFs (based on the SVAR model) to trace the net effect of a one standard deviation shock of endogenous variables on other endogenous variables, for a period of 14 days. They capture the immediate and lagged, direct and indirect interactions among the endogenous variable.

Figure 2: Response of OS to FB (± 2 S.E.s)

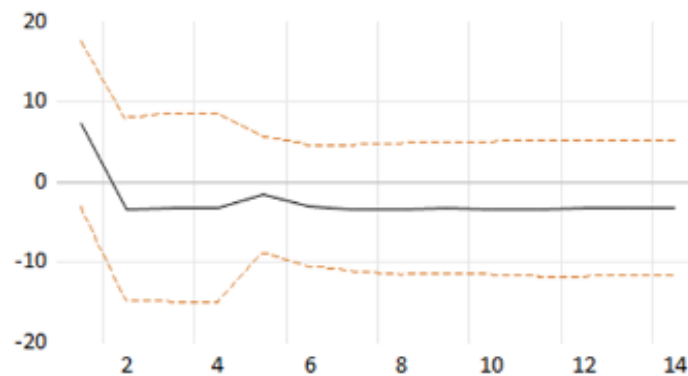


Figure 2 shows how total organic sessions responds to a one standard deviation increase in total Facebook impressions. The IRF starts with a positive response of *OS* to a shock in *FB* in the immediate next period. This uptick suggests an immediate term positive synergy. This seems to align with the expectation that paid advertising leads to an increase in organic traffic, possibly through brand awareness elevation or increased visibility. This effect is, however, not sustained. After the initial positive impact, the IRF drops to consistent negative value from period 2 through to period 14, indicating a slight but persistent negative impact of *FB* on *OS*. This could suggest that after the immediate effect wears off, increase in display ad impressions leads to a small decline in organic engagement. Users may be less inclined to search organically after being exposed to the ads, possibly due to a substitution effect where the paid impressions fulfil their information or engagement needs.

However, it is critical to note that none of the responses are statistically significant. Observe that the 95% confidence intervals include zero at all periods, indicating that we fail to reject the true impact could be zero or of an opposite sign. The immediate positive response is the closest to being statistically significant. This could indicate that the initial boost in organic traffic due to an increase in display ad impressions quickly diminishes.

Two limitations could have led to this result:

1. While the insignificance could lead us to nondefinitive conclusions, note that the wide confidence interval band could be due to a relatively small sample size to parameter ratio. An extension of the observation window could yield more definitive insights of the true dynamics between display ad impressions and organic traffic.

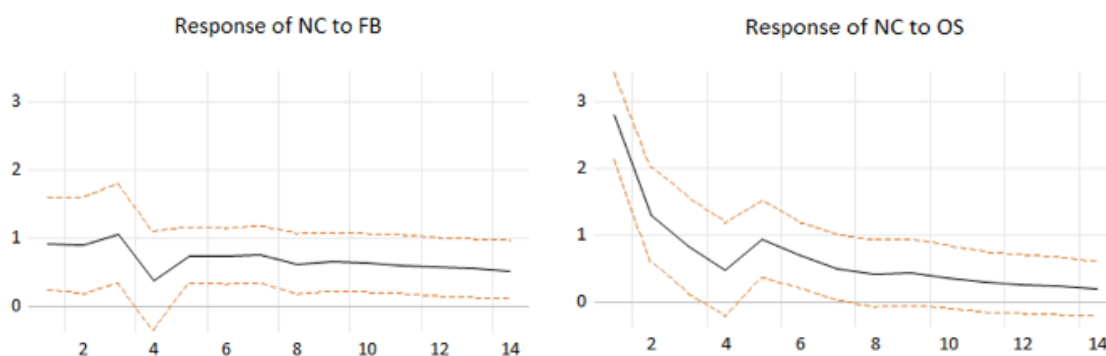
2. Recall that the optimal lag length to include in this model is determined to be 4 lags, which is based on the available data points and optimal lag selection criteria within the VAR model. However, the true temporal span necessary for the advertising effects to take effect in organic traffic could extend well beyond 4 days. For example, it could take a few weeks for users to gradually progress from initial ad exposure to active organic search. But the lag selection criteria still suggest 4 lags because they are optimised to avoid over-parameterisation and loss of degrees of freedom. A larger dataset could allow for longer lags or lower data frequency, which would be effective in addressing this lag selection issue.

With the sample size limitation in mind, there are still potential ways to assess this relationship. Additional qualitative analysis on the following could be helpful:

1. Details about the nature of the Facebook campaign content could help to explain whether the initial positive response make sense. High-quality or highly engaging content might drive organic interest in the immediate next day, even if this effect does not sustain.
2. Understanding the Click-Through Rates (CTR) of Facebook ads could help to explain the negative response of organic traffic in the following periods. High CTRs on Facebook may indicate that the ads effectively capture user attention, which could temporarily divert traffic away from organic sessions and lead to traffic cannibalisation.
3. Assessing whether there is an overlap between the target audiences could help to determine the substitution or complementarity between Facebook display ads and organic sessions.

Now we will turn to look at how paid and organic marketing individually drives new customer acquisition (ie. sales). Figure 3 shows the response of NC to OS and FB.

Figure 3: Response of NC to FB and OS (± 2 S.E.s)



The response of *NC* to *FB* shows small and yet persistent positive effect. A shock of 60,000 impressions on Facebook generates 1 new customer sign-up initially. The positive effect slowly and steadily diminishes to 0.5 new customer sign-up after 14 periods (2 weeks). This effect indicates the enduring value of Facebook ads in leading up to sales. The persistence could be explained by social media display ad's ability to have significant effects on user engagement and subsequent behaviours beyond the platform (Yousef, Dietrich and Rundle-Thiele, 2021).

The response of *NC* to *OS* starts strong and has diminishing positive effect. A shock of 227 organic sessions generates 3 new customer sign-ups initially. After a dust-settling period of 4 days, the effect settles at 0.5 new customers and slowly wanes through to the end of the 14 days. The larger initial response would reflect the high immediate impact of organic traffic on customer acquisition. This pattern could be attributed to the higher intent and proactiveness associated with customers that arrives through organic sessions, because customer-initiated touchpoints are based on their own search actions and "are considered far less intrusive" (De Haan, Wiesel and Pauwels, 2016). The diminishing effect in later periods reflects a quick decay of user engagement over a few days.

Comparing the two impulse responses, we can infer that paid display ads and organic sessions affect sales with different temporal patterns. Paid ads have a more persistent impact while organic traffic have a pronounced initial impact that quickly diminishes. Their effectiveness, however, cannot be directly compared because they represent different stages in the customer conversion funnel. Impressions are preliminary touchpoints while organic sessions are post-click, user-initiated engagement. If we have further information about the

conversion rate from Facebook impressions to clicks, it is possible to calculate and compare the effectiveness of paid and organic channels in driving conversion.

5.3 Evaluation of Model Robustness and Validity Concerns

The estimated SVAR and subsequent analysis are driven by a set of model specification assumptions. I conduct sensitivity checks to ensure robustness and validity of the model.

Temporal Robustness: A subsample analysis was conducted using only the first and the last 300 days out of the 523 days of the dataset. Both the contemporaneous effect matrix and the impulse response functions were found to be consistent to the full sample. The consistency across time periods ensures that the model is not subject to shifts in market dynamic or external events across periods.

Sensitivity to specified lag length: The SVAR model is estimated again with extended lag lengths of 7 lags and 14 lags. While the effect of *FB* on *OS* and the effect of *FB* on *NC* becomes both statistically insignificant, the sign of the contemporaneous effects remains unchanged. The insignificance could be explained by the inclusion of more parameters (87 for 7 lags and 150 for 14 lags) consuming degrees of freedom and increasing the standard errors. Despite this, the impulse response function preserves its shape and direction, indicating that the model is robust against the specification of lag length.

Sensitivity to IRF restriction: An unrestricted IRF was estimated to check for the SVAR model specification. Unrestricted IRFs trace out the impact of generalised one standard deviation shocks, and does not pre-suppose any causal ordering in the model. The unrestricted IRFs yield results that closely align with the restricted IRFs based on the SVAR restrictions. This alignment supports the appropriateness of the causal ordering (*FB* - *OS* - *NC*) used in the model, suggesting that it does accurately reflect the intrinsic ordering among the variables.

It is important to that this study faces several limitations that may affect its internal and external validity. Concerns regarding internal validity may arise in data aggregation and model specification process. The use of aggregate volume for Facebook ad impressions,

Google clicks and organic sessions may average out the distinct impact of each channel. The model is also vulnerable to omitted variable bias, as it does not account for external factors like competitor activities, market conditions and management decision that influence marketing effectiveness and customer behaviour. Additionally, using variables at their levels rather than other functional forms (eg. logarithmic transformation) may not accurately capture non-linear effects or more complex dynamics.

Regarding external validity, concerns can arise due to lack of financial and performance metrics and the lack of individual-level data. No data is available on key metrics like cost-per-click (CPC), click-through rate (CTR), and conversion rate, restricting the ability to assess the actual cost-effectiveness or synergy effect. This study thus lacks applicability in guiding real-world marketing budget allocations. Furthermore, this study relies on aggregate data, which conceals the individual-level behaviours and the specific pathways through which any synergy between paid display ads and organic traffic can happen. Consequently, while the study may indicate the presence of some synergy effects, it lacks the granularity to inform how these effects manifest or their extent. A more comprehensive dataset would be optimal for overcoming these challenges.

6. Conclusion

In summary, this study has examined the intertemporal dynamics between paid display ads and organic traffic, using a structural vector autoregression model applied to aggregate-level daily data from a small online pet food business. The key findings from this research are:

1. No Significant Synergy: The analysis found no statistically significant evidence that paid Facebook display ad impressions boost the volume of organic traffic to the business website. This finding suggests that the synergy effect highlighted in some of the existing literature may only apply under specific conditions not present in the current study context.

2. Different Sales Impact Patterns: The impact of Facebook display ads and organic traffic on new customer acquisition shows varied temporal patterns. While display ads show a sustained positive effect on sales, organic traffic exhibits a strong initial impact that quickly diminished over time. This difference suggests varying roles and effectiveness of marketing channels in customer acquisition dynamics.

The findings from this study provide insights for managers, particularly for small online enterprises. Managers should be careful in making any assumptions about channel effectiveness based on popular theories, and should instead adapt their marketing strategies based on performance data relevant to their specific market and customer base. The absence of synergy between display ads and organic traffic found in this study warns managers that they cannot rely on display ads to boost their organic traffic. Given that organic channels have the highest conversion rates, it becomes essential for businesses to independently strategize ways to enhance organic traffic. Possible organic marketing efforts include search engine optimisation, content marketing, and website UI enhancement. These strategies can help attract more organic, high-intent customer visits. The lack of synergy also raises important questions about the effectiveness of current display advertising strategies. Businesses might need to reconsider the content, placement, or creative aspects of their social media display ads, to design more integrated marketing strategies. Moreover, if display ads are not contributing to an increase in organic traffic, businesses might explore the potential for synergy between other paid channels and organic traffic, such as paid search engine

advertisements. Businesses could consider reallocating the digital marketing budget to exploit any existing synergies, optimising their overall marketing investment for better returns.

The pattern and duration of impact of Facebook display ads and organic sessions on new customer acquisition also serves as a guide to marketing decision-making. The sustained positive effect of display ads on sales suggests that these ads are crucial not just for immediate conversion but for maintaining a consistent level of consumer engagement over time. Managers should consider using display ads not only to drive direct sales but also as a tool for long-term brand reinforcement. This could involve a strategic scheduling of ad campaigns to ensure steady presence in consumer feeds, thus maintaining awareness among potential customers. Furthermore, the quick drop in impact of organic traffic on sales shows the importance of capturing and converting organic traffic effectively during its peak. To capitalise on this initial surge, businesses should ensure that their landing pages are optimised for conversion.

In conclusion, this paper provides new insights into the dynamics of digital marketing strategies in a small business context. The findings challenge some traditional assumptions about the role of paid and organic channels. By adopting a dynamic and data-driven approach to marketing budget allocation and strategy design, small businesses can better navigate the competitive online marketplace.

Appendix

Table 8: VAR lag order selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-7804.965	NA	3.25E+09	30.41540	30.63791	30.50260
1	-7275.582	1034.094	4.31E+08	28.39449	28.69117	28.51076
2	-7242.814	63.62860	3.93E+08	28.30219	28.67304*	28.44753*
3	-7226.422	31.63769	3.82E+08	28.27348	28.71850	28.44789
4	-7209.922	31.65410*	3.71e+08*	28.24436*	28.76355	28.44783
5	-7207.081	5.417784	3.8E+08	28.26827	28.86163	28.50081
6	-7199.990	13.43807	3.83E+08	28.27569	28.94322	28.53729
7	-7192.500	14.10675	3.85E+08	28.28155	29.02325	28.57223
8	-7186.416	11.38944	3.89E+08	28.29288	29.10875	28.61262

System of equations estimated in the VAR Model:

$$\begin{aligned}
 FB = & C(1) * FB(-1) + C(2) * FB(-2) + C(3) * FB(-3) + C(4) * FB(-4) + C(5) * OS(-1) \\
 & + C(6) * OS(-2) + C(7) * OS(-3) + C(8) * OS(-4) + C(9) * NC(-1) + C(10) \\
 & * NC(-2) + C(11) * NC(-3) + C(12) * NC(-4) + C(13) + C(14) \\
 & * GCLICKS_ADJ + C(15) * (@WEEKDAY = 2) + C(16) * (@WEEKDAY \\
 = & 3) + C(17) * (@WEEKDAY = 4) + C(18) * (@WEEKDAY \\
 = & 5) + C(19) * (@WEEKDAY = 6) + C(20) * (@WEEKDAY \\
 = & 7) + C(21) * @TREND
 \end{aligned}$$

$$\begin{aligned}
 OS = & C(22) * FB(-1) + C(23) * FB(-2) + C(24) * FB(-3) + C(25) * FB(-4) + C(26) \\
 & * OS(-1) + C(27) * OS(-2) + C(28) * OS(-3) + C(29) * OS(-4) + C(30) \\
 & * NC(-1) + C(31) * NC(-2) + C(32) * NC(-3) + C(33) * NC(-4) + C(34) \\
 & + C(35) * GCLICKS_ADJ + C(36) * (@WEEKDAY = 2) + C(37) * (@WEEKDAY \\
 = & 3) + C(38) * (@WEEKDAY = 4) + C(39) * (@WEEKDAY \\
 = & 5) + C(40) * (@WEEKDAY = 6) + C(41) * (@WEEKDAY \\
 = & 7) + C(42) * @TREND
 \end{aligned}$$

$$\begin{aligned}
 NC = & C(43) * FB(-1) + C(44) * FB(-2) + C(45) * FB(-3) + C(46) * FB(-4) + C(47) \\
 & * OS(-1) + C(48) * OS(-2) + C(49) * OS(-3) + C(50) * OS(-4) + C(51) \\
 & * NC(-1) + C(52) * NC(-2) + C(53) * NC(-3) + C(54) * NC(-4) + C(55) \\
 & + C(56) * GCLICKS_ADJ + C(57) * (@WEEKDAY = 2) + C(58) * (@WEEKDAY \\
 = & 3) + C(59) * (@WEEKDAY = 4) + C(60) * (@WEEKDAY \\
 = & 5) + C(61) * (@WEEKDAY = 6) + C(62) * (@WEEKDAY \\
 = & 7) + C(63) * @TREND
 \end{aligned}$$

Table 9: Detailed VAR Results by Variable

	Coefficient	Std. Error	t-Statistic	Prob.
<i>C</i> (1)	0.687737	0.044856	15.33223	0.0000
<i>C</i> (2)	0.118403	0.054659	2.166191	0.0305
<i>C</i> (3)	-0.015584	0.054649	-0.285162	0.7756
<i>C</i> (4)	0.120371	0.045185	2.663983	0.0078
<i>C</i> (5)	-0.005298	0.008605	-0.615666	0.5382
<i>C</i> (6)	0.001416	0.009155	0.154655	0.8771
<i>C</i> (7)	0.004277	0.009150	0.467443	0.6403
<i>C</i> (8)	-0.004358	0.008554	-0.509495	0.6105
<i>C</i> (9)	0.096182	0.130454	0.737286	0.4611
<i>C</i> (10)	0.136106	0.130913	1.039670	0.2987
<i>C</i> (11)	-0.079832	0.130553	-0.611492	0.5410
<i>C</i> (12)	-0.053134	0.128980	-0.411956	0.6804
<i>C</i> (13)	10.32231	5.723775	1.803409	0.0715
<i>C</i> (14)	0.014019	0.008507	1.648001	0.0996
<i>C</i> (15)	-9.003498	4.019383	-2.240020	0.0252
<i>C</i> (16)	-6.987666	4.492359	-1.555456	0.1200
<i>C</i> (17)	-9.235251	4.470530	-2.065807	0.0390
<i>C</i> (18)	-10.57209	4.170631	-2.534890	0.0113
<i>C</i> (19)	4.954626	4.055899	1.221585	0.2221
<i>C</i> (20)	8.747383	3.986313	2.194354	0.0284
<i>C</i> (21)	0.010145	0.010518	0.964514	0.3349
<i>C</i> (22)	-0.305570	0.248981	-1.227284	0.2199
<i>C</i> (23)	0.061525	0.303398	0.202788	0.8393
<i>C</i> (24)	0.003553	0.303338	0.011713	0.9907
<i>C</i> (25)	0.044299	0.250807	0.176625	0.8598
<i>C</i> (26)	0.402169	0.047761	8.420371	0.0000
<i>C</i> (27)	0.133380	0.050819	2.624621	0.0088
<i>C</i> (28)	0.087493	0.050790	1.722648	0.0852
<i>C</i> (29)	0.130207	0.047481	2.742323	0.0062
<i>C</i> (30)	0.174009	0.724115	0.240306	0.8101
<i>C</i> (31)	0.198238	0.726661	0.272807	0.7850
<i>C</i> (32)	0.496830	0.724662	0.685602	0.4931
<i>C</i> (33)	0.149941	0.715932	0.209435	0.8341
<i>C</i> (34)	151.2354	31.77099	4.760173	0.0000
<i>C</i> (35)	0.021453	0.047219	0.454325	0.6497
<i>C</i> (36)	74.50278	22.31041	3.339373	0.0009
<i>C</i> (37)	26.24237	24.93576	1.052399	0.2928
<i>C</i> (38)	41.88352	24.81460	1.687858	0.0916
<i>C</i> (39)	-34.03560	23.14995	-1.470224	0.1417
<i>C</i> (40)	-199.4012	22.51310	-8.857119	0.0000
<i>C</i> (41)	-65.90825	22.12685	-2.978654	0.0029
<i>C</i> (42)	0.218666	0.058382	3.745450	0.0002
<i>C</i> (43)	0.031795	0.016342	1.945639	0.0519
<i>C</i> (44)	0.016309	0.019913	0.818985	0.4129
<i>C</i> (45)	-0.031147	0.019909	-1.564436	0.1179

<i>C</i> (46)	0.009176	0.016461	0.557453	0.5773
<i>C</i> (47)	0.006242	0.003135	1.991252	0.0466
<i>C</i> (48)	-0.000708	0.003335	-0.212260	0.8319
<i>C</i> (49)	-0.003360	0.003334	-1.007834	0.3137
<i>C</i> (50)	0.000417	0.003116	0.133847	0.8935
<i>C</i> (51)	0.191943	0.047526	4.038651	0.0001
<i>C</i> (52)	0.123468	0.047694	2.588786	0.0097
<i>C</i> (53)	0.134152	0.047562	2.820551	0.0049
<i>C</i> (54)	0.164875	0.046989	3.508759	0.0005
<i>C</i> (55)	3.745619	2.085255	1.796241	0.0727
<i>C</i> (56)	0.010397	0.003099	3.354718	0.0008
<i>C</i> (57)	-1.696010	1.464320	-1.158224	0.2470
<i>C</i> (58)	-2.445741	1.636632	-1.494374	0.1353
<i>C</i> (59)	-1.020387	1.628679	-0.626512	0.5311
<i>C</i> (60)	-4.004289	1.519421	-2.635404	0.0085
<i>C</i> (61)	-4.574855	1.477623	-3.096091	0.0020
<i>C</i> (62)	-2.608131	1.452272	-1.795897	0.0727
<i>C</i> (63)	0.008828	0.003832	2.303966	0.0214

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